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INTERACTION OF DISCONTINUITIES IN MAGNETIZABLE IDEALLY-CONDUCTING INCOMPRESSIBLE MEDIA

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Interaction of the discontinuities (shock waves S, contact discontinuities K, Alfvén discontinuities A) in magnetizable ideally-conducting incompressible medium, are studied. At the instant of interaction, a discontinuity arises which will undergo disintegration and on which, generally speaking, the conservation laws do not hold. Possible combinations of waves and discontinuities formed at disintegration are determined.

The interaction of normal gas dynamic waves and discontinuities was studied in [1], while [2 — 4] dealt with the interaction of magnetohydrodynamic waves and discontinuities.

We assume that the medium under consideration is described by the following equations of state: $\rho = \text{const}$, $V_0 = c_v T + \text{const}$, $M = K(\Theta - T)$. In the course of solution we use the results obtained and the notation obtained in [5]. As in [5], we assume that $B_n^2 / (4\pi\rho c_v \Theta) \ll 1$ and $4\pi K\Theta / B_n \lesssim 1$.

The parameters of the medium between interacting discontinuities prior to collision will be denoted by the index 1, the parameters of the medium at the instant of collision to the left and right of the plane of collision will be denoted by the index 0, and those of the medium to the left of the plane of interaction between the discontinuities will be denoted by a prime.

1. Interaction of shock waves. Let us consider a collision of two shock waves of arbitrary intensity. In [5] we have studied the behavior of the parameters in the shock waves propagating through magnetizable ideally conducting incompressible media. It was shown that such waves are plane polarized, and that the problem of interaction of two shock waves can be regarded as a plane problem. Let us draw in the plane $v_\tau H_\tau$ from the point A with coordinates $v_{\tau 1}$, $H_{\tau 1}$ the lines corresponding to shock waves propagating to the left and right through the medium the parameters of which are denoted by the index 1 (Fig. 1).

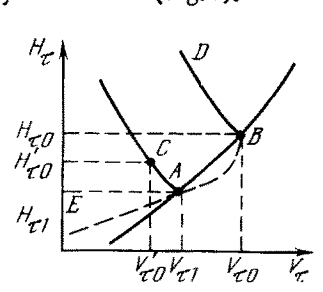

Fig. 1

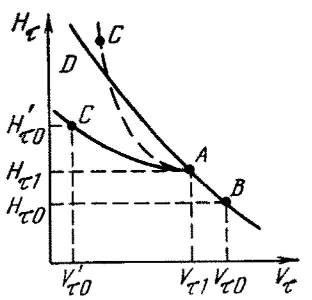

Fig. 2

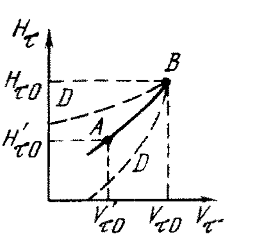

Fig. 3

The directions in which the shock waves propagate are indicated in Fig. 1 by arrows. The point B (C) with coordinates $v_{\tau 0}$, $H_{\tau 0}$ ($v_{\tau 0}'$, $H_{\tau 0}'$) lies on the line corresponding to the shock wave moving to the left (right) through the medium with parameters $v_{\tau 1}$, $H_{\tau 1}$, T_1.

The problem of collision of two shock waves reduces to the problem of disintegration of an arbitrary discontinuity (the plane case) which separates the medium with the parameters $v_{\tau 0}$, $H_{\tau 0}$ (point B in Fig. 1) from the medium with the parameters $v_{\tau 0}'$, $H_{\tau 0}'$ (point C in Fig. 1).

Let us draw from the point B a line BD corresponding to the shock wave moving to the right through the medium with parameters $v_{\tau 0}$, $H_{\tau 0}$. This line corresponds to the combination KS. Using the results of [5] we can show that the lines DB and CA do not intersect each other. Indeed, the equation determining their point of intersection has the form

$$v_{\tau 1} - v_{\tau 0} - (H_\tau - H_{\tau 1})\Psi(H_\tau, H_{\tau 1}) + (H_\tau - H_{\tau 0})\Psi(H_\tau, H_{\tau 0}) = 0 \tag{1.1}$$

Since $(H_\tau - H_{\tau a})\Psi(H_\tau, H_{\tau a})$ is a monotonously decreasing function of $H_{\tau a}$ and $H_{\tau 1} < H_{\tau 0}$, $v_{\tau 1} < v_{\tau 0}$; Eq. (1.1) has no roots. Moreover, the line BE corresponding to the wave combination SK and representing the geometrical locus of points from which one can reach the point B with the help of the shock wave to the left, passes through the point A. The line BE is depicted in Fig. 1 with a dashed line.

Thus the point C reaches the region situated above the line BE corresponding to the combination SK, and $H_\tau = 0$ and lies below the line BD corresponding to the combination KS (Fig. 1). This region corresponds to the combination of waves SKS. Consequently, a collision of two shock waves generates another two shock waves moving to the left and right. The absolute magnitude of the magnetic field vector in the region between these shock waves increases compared with the magnetic fields $H_{\tau 1}$, $H_{\tau 0}$, $H_{\tau 0}'$.

Let us consider the interaction of two shock waves moving in the same direction and one overtaking the other. We assume for definiteness that both shock waves move to the right. The distribution of the points A ($v_{\tau 1}$, $H_{\tau 1}$), B ($v_{\tau 0}$, $H_{\tau 0}$), C ($v_{\tau 0}'$, $H_{\tau 0}'$) shown in Fig. 2 differs from the previous one. In the general case the line AC (shown in Fig. 2 by dashed line) can intersect the line BAD corresponding to the combination of waves KS, and the point C with coordinates $v_{\tau 0}'$, $H_{\tau 0}'$ may lie above, as well as below the line BAD. Thus in the case of one shock wave overtaking the other we obtain either two shock waves moving in the different directions, or a shock wave moving to the right through the medium with parameters $v_{\tau 0}$, $H_{\tau 0}$ and a centered wave moving to the left through the medium with parameters $v_{\tau 0}'$, $H_{\tau 0}'$.

In the present case we have, as was shown in [5], $(B_n^2 / (4\pi\rho c_v \Theta) \ll 1$, $4\pi K\Theta / B_n \lesssim 1)$, the temperature jump $\{\tau\} \ll 1$, $\tau = T / \Theta$, and we can assume that the shock waves do not affect the temperature distribution in the medium ($T_0 = T_0' = T_1$). We shall assume that the waves appearing in the process of disintegration do not affect the temperature distribution either, and from this it follows that the interaction of the shock waves does not give rise to contact discontinuities as the temperature is the only parameter undergoing a jump.

2. Interaction of the contact discontinuity with the shock wave. Let us consider a collision between a shock wave and a contact discontinuity. We shall consider, for definiteness, the case when the shock wave approaches the contact discontinuity from the right direction. At the contact discontinuity the temperature undergoes an arbitrary jump $\{T\} = T_1 - T_0' \neq 0$. When the shock wave collides with the contact discontinuity, a discontinuity appears between the media with parameters $v_{\tau 0}, H_{\tau 0}$, T_0 and $v_{\tau 0}'$, $H_{\tau 0}'$, T_0'. The parameters v_τ and H_τ of the medium lying to the left and right of the collision-generated discontinuity lie on the shock adiabate which originates at the point A with coordinates $v_{\tau 0}'$, $H_{\tau 0}'$ and moving to the point B with coordinates $v_{\tau 0}$, $H_{\tau 0}$ (Fig. 3). The equation of this line represents an equation of the shock adiabate corresponding to the shock wave propagating to the left through the medium with parameters $v_{\tau 0}'$, $H_{\tau 0}'$, T_0. From this it follows that the point A need not, generally speaking, lie on the line BD which is the geometrical locus of the points from which one can reach the point B $(v_{\tau 0}, H_{\tau 0})$ along the shock adiabate corresponding to the shock wave propagating to the left through the medium with parameters $v_{\tau 0}'$, $H_{\tau 0}'$, T_0' (two possible positions of this line relative to the point A are shown by dashed lines in Fig. 3).

When the point A lies below the line BD (Fig. 3), the interaction of the shock wave with a contact discontinuity generates a shock wave moving to the left through the medium with parameters $v_{\tau 0}'$, $H_{\tau 0}'$, T_0' , a centered wave moving to the right through the medium with parameters $v_{\tau 0}$, $H_{\tau 0}$, T_0 , and a contact discontinuity separating these waves (combination SKR).

When the point A lies above the line BD (Fig. 3), the interaction of the shock wave with a contact discontinuity generates two shock waves moving through the medium with parameters $v_{\tau 0}'$, $H_{\tau 0}'$, T_0' $(v_{\tau 0}, H_{\tau 0}, T_0)$ to the left (right) and separated by the contact discontinuity (combination $\overset{\leftarrow}{S}K\vec{S}$). If the shock wave approaches the contact discontinuity from the left, it passes through it with diminished intensity and, depending on the temperature jump at the contact discontinuity, a reflected shock wave or a centered wave may appear.

3. Interactions of the shock waves and the Alfvén discontinuities. Consider a collision of a shock wave and an Alfvén discontinuity. We assume, for definiteness, that the Alfvén discontinuity propagates to the right and the shock wave to the left, through the medium with parameters $\mathbf{v}_{\tau 1}$, $\mathbf{H}_{\tau 1}$, T_1.

From the relations at the Alfvén discontinuity and the shock wave it follows that

$$|\mathbf{H}_{\tau 0}| > |\mathbf{H}_{\tau 0}'|, \quad T_0' = T_0 \tag{3.1}$$

In [5] we have shown that when the condition (3.1) holds, an arbitrary discontinuity may disintegrate into the following combinations: $RAAR$, $SAAR$, $SAAS$ (no contact discontinuity occurs), depending on the region of the plane Δv, Δw in which the point $\Delta v_0 = v_0 - v_0'$, $\Delta w_0 = w_0 - w_0'$ falls (here v and w are the projections of the velocity $\mathbf{v}_\tau$ on the y- and z-axes). The y-axis and z-axis lie in the plane of the discontinuity generated. It can be shown that the combination $RAAR$ of waves cannot arise from a collision of the Alfvén discontinuity with a shock wave.

The plane Δv, Δw is divided by the circles corresponding to the combinations SAA and AAR into three regions: $SAAS$, $SAAR$ and $RAAR$. The region in which the combination $RAAR$ is realized is bounded by the circle AAR given by the following equation:

$$(\Delta v_\tau - \mathbf{L})^2 = R^2, \quad \mathbf{L} = X(|\mathbf{H}_{\tau 0}'|, |\mathbf{H}_{\tau 0}|)\mathbf{H}_{\tau 0}/|\mathbf{H}_{\tau 0}| - a_{A0}'\mu_0(\mathbf{H}_{\tau 0}' + \mathbf{H}_{\tau 0}|\mathbf{H}_{\tau 0}'|/|\mathbf{H}_{\tau 0}|)/B_n, \quad R = 2a_{A0}'\mu_0'|\mathbf{H}_{\tau 0}'|/B_n \tag{3.2}$$

From the relations on the shock wave and the Alfvén discontinuity it follows, that

$$\Delta v_{\tau 0} = \mathbf{v}_{\tau 0} - \mathbf{v}_{\tau 0}' = \Psi(|\mathbf{H}_{\tau 0}|, |\mathbf{H}_{\tau 0}'|)(|\mathbf{H}_{\tau 0}| - |\mathbf{H}_{\tau 0}'|)\frac{\mathbf{H}_{\tau 0}}{|\mathbf{H}_{\tau 0}|} - \frac{a_{A0}'\mu_0'}{B_n}\left(\mathbf{H}_{\tau 0}' - \mathbf{H}_{\tau 0}\frac{|\mathbf{H}_{\tau 0}'|}{|\mathbf{H}_{\tau 0}|}\right) \tag{3.3}$$

Clearly, the point $\Delta v_{\tau 0}$ is outside the circle (3.2) since the inequality

$$|\mathbf{L} - \Delta v_{\tau 0}| = |\Psi(|\mathbf{H}_{\tau 0}|, |\mathbf{H}_{\tau 0}'|)(|\mathbf{H}_{\tau 0}| - |\mathbf{H}_{\tau 0}'|) - X(|\mathbf{H}_{\tau 0}'|, |\mathbf{H}_{\tau 0}|)\frac{\mathbf{H}_{\tau 0}}{|\mathbf{H}_{\tau 0}|} + \frac{2a_{A0}'\mu_0'}{B_n}\mathbf{H}_{\tau 0}'| > R \tag{3.4}$$

holds. The inequality (3.4) always holds, since $X(|\mathbf{H}_{\tau 0}'|, |\mathbf{H}_{\tau 0}|) < 0$ and $\Psi(H_{\tau 0}, H_{\tau 0}')(H_{\tau 0} - H_{\tau 0}') > 0$. Thus, when a shock wave collides with an Alfvén discontinuity, either the combination $SAAS$ or the combination $SAAR$ may arise.

Consider the case when the shock wave overtakes the Alfvén discontinuity. We assume, for definiteness, that the Alfvén discontinuity and the shock wave both propagate to the right. From the relations at the Alfvén discontinuity and the shock wave it follows that

$$|\mathbf{H}_{\tau 0}'| > |\mathbf{H}_{\tau 0}|, \quad T_0 = T_0' \tag{3.5}$$

We have shown in [5] that generally, when the condition (3.5) holds, the following combinations of waves and discontinuities can be realized: $RAAR$, $RAAS$ and $SAAS$, and a contact discontinuity does not appear. As in the case of collision of the shock wave and the Alfvén discontinuity, it can be shown that in the present case we have either the combinations $RAAS$ or $SAAS$. The combination $RAAR$ does not appear irrespective of the intensities of the shock waves and the Alfvén discontinuities. When the Alfvén discontinuity and the shock wave both move to the left, their interaction generates either the combination $SAAS$ or the combination $SAAR$ of waves and discontinuities.

4. Interaction of Alfvén discontinuities. Let us consider the interaction of rotating Alfvén discontinuities of arbitrary intensity. The parameters of the medium to the left and right of the discontinuity appearing as the result of the collision of two Alfvén discontinuities of arbitrary intensity, are connected by the following relations:

$$\mathbf{v}_{\tau 0} - \mathbf{v}_{\tau 0}' - \frac{a_{A0}\mu_0}{B_n}(\mathbf{H}_{\tau 0} + \mathbf{H}_{\tau 0}') = -\frac{2\mu_0 a_{A0}\mathbf{H}_{\tau 1}}{B_n} \tag{4.1}$$

$$|\mathbf{H}_{\tau 1}| = |\mathbf{H}_{\tau 0}| = |\mathbf{H}_{\tau 0}'|, \quad T_0 = T_0', \quad a_{A0} = a_{A0}'$$

Let the angle between the vectors $\mathbf{H}_{\tau 0}$ and $\mathbf{H}_{\tau 0}'$ be equal to γ. This angle determines the difference between the intensities of the colliding discontinuities. From the first equation of (4.1) it follows that $\Delta v_{\tau 0} = \mathbf{v}_{\tau 0} - \mathbf{v}_{\tau 0}'$ lies on the circle δ of radius

$$2a_{A0}\mu_0 H_{\tau 0} / B_n$$

in the Δv, Δw-plane with the center at the point Δv^{00}, Δw^{00}, where

$$\Delta v^{00} = \frac{a_{A0}\mu_0}{B_n} H_{\tau 0}(1 + \cos\gamma), \quad \Delta w^{00} = \frac{a_{A0}\mu_0}{B_n} H_{\tau 0}\sin\gamma \tag{4.2}$$

The y-axis coincides with the direction of $\mathbf{H}_{\tau 0}$.

To show which combination of waves and discontinuities the initial discontinuity generated by the collision of two Alfvén discontinuities disintegrates into, we turn to the results of [5]. Since $|\mathbf{H}_{\tau 0}| = |\mathbf{H}_{\tau 0}'|$, $T_0 = T_0'$, three combinations of waves and discontinuities may be realized: $RAAR$, AA, $SAAS$. The combinations $RAAR$ and $SAAS$ correspond in the Δv, Δw-plane to the regions inside and outside the circle σ, the latter corresponding to the combination AA. The radius of σ is equal to the radius of δ and the coordinates of its center are $\Delta v^0 = \Delta v^{00}$, $\Delta w^0 = -\Delta w^{00}$ (see formulas (4.2)). The collision of two Alfvén discontinuities does not generate a contact discontinuity.

The circles σ and δ may intersect each other at $-180° < \gamma < 180°$, $\gamma \neq 0$, touch each other when $\gamma = 0$ and coincide with each other when $\gamma = \pm 180°$. Consequently, when two Alfvén discontinuities collide, the following combinations can be realized depending on the angle γ and on the particular values of the velocity differences $\Delta v_{\tau 0} = \mathbf{v}_{\tau 0} - \mathbf{v}_{\tau 0}'$: (1) $SAAS$; $\gamma = 0$; (2) $SAAS$, $RAAR$, AA; $-180° < \gamma < 180°$; (3) AA; $\gamma = \pm 180°$.

5. Interaction of the Alfvén and the contact discontinuities. Consider a collision of an Alfvén discontinuity of intensity $\gamma \neq 0$, with a contact discontinuity. We shall assume for definiteness that the Alfvén discontinuity moves towards the contact discontinuity from the right. From the relations at the Alfvén discontinuity and the contact discontinuity it follows that

$$|\mathbf{H}_{\tau 0}'| = |\mathbf{H}_{\tau 0}|, \quad \{T\} = T_0' - T_0 \neq 0 \tag{5.1}$$

It was shown in [5] that when the conditions (5.1) hold, the initial discontinuity connecting the media with parameters $\mathbf{v}_{\tau 0}$, $\mathbf{H}_{\tau 0}$, T_0 and $\mathbf{v}_{\tau 0}'$, $\mathbf{H}_{\tau 0}'$, T_0' can generally disintegrate into the following combinations: $SAKAS$, AKA and $RAKAR$.

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